THE GAME OF BILLIARDS

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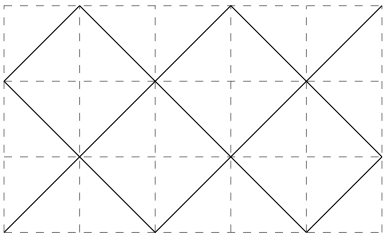
You are given a billiards table of length 5 and breadth 3. If you shoot a ball from one of the corners (for now, assume this is bottom right corner) at 45°, what is the distance travelled by the ball travel before falling into a hole (reaching one of the corners)?

Notice that drawing a grid as given below would help you answer the question:



1. Explore the same question using tables of different dimensions:

   (a) 2×3 (b) 5×7 (c) 3×6 (d) 4×7

2. Try to find some pattern and make conjectures. Try to justify your claims. Let the following questions guide you:

   (a) Does the parity (odd or even) of the dimensions (length or breadth) have any significance?

   (b) Does the primality (whether prime or not) of the dimensions (length or breadth) have any significance?

   (c) Is there some relationship between the length and breadth of importance?

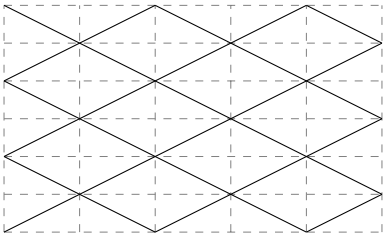
3. Would the answers change if we started from some corner other than the bottom right corner?

4. Did the ball always pass through all squares in the grid? What relation ship between the length and breadth would guarantee that the ball passess through all squares?

5. Given the length and breadth of the billiards table, can you predict which hole the ball would fall in?

6. Given the length and breadth of the billiards table, can you predict can you predict how many times the ball would bounce before it falls in one of the holes?

7. We may not always shoot the ball at 45°. Notice that by forming a slightly different grid (namely a 5×6 gird) for the 5×3 billiards ball, I can study what happens when I shoot the ball at a different angle. What is the angle at which I have shot the ball?



8. What are the angles that can be studied using the above technique? **Hint: What values can** $\tan(\theta)$ **take?**
9. Can you plot the point $(\frac{2}{3}, \frac{7}{4})$ on the 5×3 table? Can you find the trajectory of the ball if you shoot it from $(\frac{2}{3}, \frac{7}{4})$ at an angle 45°? **Hint: Draw an appropriate grid**
10. Can you generalise the above ideas to study what happens when you shoot the ball from a point with co-ordinates (a, b) where a and b are rationals? Further generalise by letting the angle to be any θ, where $\tan(\theta)$ takes "apporopriate values".